PG – 120

IV Semester M.Sc. Examination, June 2015
(Scheme RNS)
MATHEMATICS
M 404A : Magnetohydrodynamics

Time : 3 Hours Max. Marks : 80

Instructions : *i) Answer any five questions*
ii) All questions carry equal marks

1 a) Explain the emu and esu systems of units and thereby explain the need for the SI system of units. 6

b) Derive any two fundamental laws of electromagnetics. 10

2 a) With the usual notation, prove that $\nabla \cdot \vec{P} = -\rho_{eb}$ 6

b) Prove or disprove the following : 10

i) The normal component of electric field is discontinuous across an interface.

ii) The tangential component of magnetic field is continuous across an interface.

3 a) Derive the magnetic induction equation in the dimensionless form 12

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{q} \times \vec{B} \right) + \frac{1}{R_m} \nabla^2 \vec{B}.$$

Discuss the nature of this equation for $R_m << 1$ and $R_m >> 1$.

b) Discuss the boundary conditions on velocity and temperature. 4

4 a) Establish that there is no leakage of magnetic flux in an ideal fluid. 9

b) Prove that $\frac{D}{Dt}\left(\frac{\vec{H}}{\rho}\right) = \left(\frac{\vec{H}}{\rho} \cdot \nabla\right)\vec{q}$, where the quantities have their usual meaning.

Stating the assumptions, further show that $\frac{\vec{H}}{\rho}$ = const. for a travelling fluid element. 7

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5. a) State and prove Chandrasekhar's theorem on a force-free magnetic field. 8

b) In the magnetostatic configuration with an axisymmetric poloidal magnetic field, show that the magnetic stream function ϕ satisfies the equation

$$\frac{\partial(\psi, \Lambda\phi)}{\partial(r, z)} = 0, \text{ where } \Lambda\phi = r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial r}\right) + \frac{\partial^2\phi}{\partial z^2} \quad 8$$

6. a) Write a short note on the following: 6

i) Longitudinal and transverse waves

ii) Dispersive and non-dispersive waves.

b) Describe briefly the experiments of Lundquist and Lehnert to demonstrate the existence of Alfven waves. 4

c) Show that the Lorentz force can be expressed as a surface force represented by the stress tensor $\overleftrightarrow{T}M$ in the form 6

$$\vec{J} \times \vec{B} = \nabla \cdot \overleftrightarrow{T}M$$

7. a) Obtain Walen's relations in their standard form. 6

b) Establish the transverse wave equations for a compressible non-viscous perfectly electrically conducting fluid in the presence of a uniform magnetic field making an angle θ with the horizontal x-axis. 10

8. Obtain the velocity and temperature distributions for a hydromagnetic plane couette flow. Discuss the effect of magnetic field on the same. 16